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LOCAL CONVERGENCE OF A HANSEN-PATRICK-LIKE FAMILY OF OPTIMAL FOURTH ORDER METHODS

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Abstract. We present a local convergence analysis of an optimal fourth order Hansen-Patrick-like family of methods in order to approximate a solution of a nonlinear equation. Earlier studies use hypotheses involving derivatives up to the third order to show convergence although only the first derivative appears in these methods. In the present study we use only hypotheses on the first derivative. We also provide computable error bounds on the distances involved based on Lipschitz constants. This way we expand the applicability of these methods. Numerical examples are also given in this study.

Keywords: Hansen-Patrick-like family, optimal method, local convergence.

AMS Subject Classification: 65D10, 65D99.

1. Introduction

In this study we are concerned with the problem of approximating a locally unique solution x^* of equation

$$F(x) = 0, \tag{1}$$

where $F : D \subseteq S \to S$ is a nonlinear function, D is a convex subset of S and S is $\mathbb{R}$ or $\mathbb{C}$. Newton-like methods are used for finding solutions of (1). These methods are usually studied based on semi-local and local convergence. The semi-local convergence matter is, based on the information around an initial point, to give conditions ensuring the convergence of the iterative procedure; while the local one is, based on the information around a solution, to find estimates of the radii of convergence balls [1]–[21].

We present the local convergence analysis of the two-step fourth order Hansen-Patrick-like family of methods defined for each $n = 0, 1, 2, \cdots$ by

$$\begin{aligned} y_n &= x_n - F\prime(x_n)^{-1}F(x_n) \\ x_{n+1} &= x_n - B_nF'(x_n)^{-1}F(x_n), \end{aligned} \tag{2}$$

where x_0 is an initial point, $\alpha \neq -1$ is a parameter,

$$A_n = \frac{1-(\alpha+3)\frac{F(y_n)}{F(x_n)} - (\alpha^2-1)(\frac{F(y_n)}{F(x_n)})^2}{1+(\alpha-1)\frac{F(y_n)}{F(x_n)}}$$

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and $B_n = \frac{\alpha+1}{\alpha \pm \sqrt{A_n}}$. Methods (2) is an alternative to Hansen-Patrick family (2) of methods defined for each $n = 0, 1, 2, \cdots$ by

$$x_{n+1} = x_n - \frac{\alpha+1}{\alpha \pm \sqrt{1-(\alpha+1)C_n}} F'(x_n)^{-1} F(x_n), \tag{3}$$

where $C_n = F'(x_n)^{-1} F''(x_n) F'(x_n)^{-1} F(x_n)$. Note that method (3) requires the computation of $F''(x_n)$ at each step where as method (2) uses only the first derivative $F'(x_n)$. The fourth order of convergence of method (2) was shown by Kanwar in [14] using Taylor expansions and hypotheses reaching up to the third derivative of function F. Method (2) reduces to: Ostrowski's square root method for $\alpha = 0$; Euler's method for $\alpha = 1$; Laquerre's method for $\alpha = \frac{1}{\mu-1}$ and as a limiting case; Halley's method.

Moreover, method (2) is an optimal fourth-order method in the sense of Kung-Traub [20]. The advantages of method (2) over other fourth order methods were given in [14]. However, the hypotheses on the third derivative limit the applicability of this method. As a motivational example, let us define function f on $D = [-\frac{1}{2}, \frac{5}{2}]$ by

$$f(x) = \begin{cases} x^3 \ln x^2 + x^5 - x^4, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Choose $x^* = 1$. We have that

$$\begin{aligned} f'(x) &= 3x^2 \ln x^2 + 5x^4 - 4x^3 + 2x^2, \; f'(1) = 3, \\ f''(x) &= 6x \ln x^2 + 20x^3 - 12x^2 + 10x \\ f'''(x) &= 6 \ln x^2 + 60x^2 - 24x + 22. \end{aligned}$$

Then, obviously, function f''' is unbounded on D. In the present paper we only use hypotheses on the first Fréchet derivative. This way we expand the applicability of method (2).

The rest of the paper is organized as follows. In Section 2 the local convergence analysis of method (2) is given. The numerical examples are presented in the concluding Section 3.

2. Local convergence analysis

We present the local convergence analysis of method (2) in this section. Let $L_0 > 0, L > 0, M \geq 1, \alpha \in S$ with $|\alpha| > 2M+1$ be parameters. It is convenient for the local convergence analysis that follows to define some functions and parameters.Define functions g_1, p_0 and h_{p_0} on the interval $[0, \frac{1}{L_0})$ by

$$\begin{aligned} g_1(t) &= \frac{Lt}{2(1-L_0 t)}, \\ p_0(t) &= \frac{|\alpha-1| M g_1(t)}{1 - \frac{L_0}{2} t}, \\ h_{p_0}(t) &= p_0(t) - 1, \end{aligned}$$

and parameter r_1 by

$$r_1 = \frac{2}{2L_0 + L}.$$

We have $h_{p_0}(0) = -1 < 0$ and $h_{p_0}(t) \to +\infty$ as $t \to (\frac{1}{L_0})^-$. It then follows by the Intermediate Value Theorem that function h_{p_0} has zeros in the interval $(0, \frac{1}{L_0})$. Denote by r_{p_0} the smallest such zero. Moreover, define functions p and h_p on the interval $[0, r_{p_0})$ by

$$p(t) \quad = \quad \frac{1}{|\alpha|}\sqrt{\frac{1+\frac{|\alpha+3|Mg_1(t)}{1-\frac{L_0}{2}t}+\frac{|\alpha^2-1|M^2g_1(t)^2}{(1-\frac{L_0}{2}t)^2}}{1-p_0(t)}}$$

and

$$h_p(t) \quad = \quad p(t)-1.$$

We get that $h_p(0)=\frac{1}{|\alpha|}-1<0$ (by the choice of $|\alpha|$) and $h_p(t)\to+\infty$ as $t\to r_{p_0}^-$. Denote by r_p the smallest such zero. Furthermore, define functions g_2 and h_2 on the interval $[0, r_p)$ by

$$g_2(t)=\frac{1}{2(1-L_0t)}(Lt+\frac{2M(1+|\alpha|p(t))}{|\alpha|(1-p(t))})$$

and

$$h_2(t)=g_2(t)-1.$$

We obtain $h_2(0)=\frac{2M(1+|\alpha|p(0))}{|\alpha|(1-p(0))}-1=\frac{2M}{|\alpha|(1-\frac{1}{|\alpha|})}-1<0$ (by the choice of $|\alpha|$) and $h_2(t)\to+\infty$ as $t\to r_p^-$. Denote by r_2 the smallest zero of function h_2 in the interval $(0, r_p)$. Set

$$r=\min\{r_1, r_2\}. \tag{4}$$

Then, we have that

$$0<r\le r_1, \tag{5}$$

and for each $t\in[0, r)$

$$0\le g_1(t)<1, \tag{6}$$

$$0\le p_0(t)<1, \tag{7}$$

and

$$0\le p(t)<1, \tag{8}$$

and

$$0\le g_2(t)<1. \tag{9}$$

Let $U(v,\rho), \bar{U}(v,\rho)$ stand for the open and closed balls in S, respectively, with center $v\in S$ and of radius $\rho>0$. Next, we present the local convergence analysis of method (2) using the preceding notation.

Theorem 2.1. *Let $F: D\subseteq S\to S$ be a differentiable function. Suppose that there exist $x^*\in D$, parameters $L_0>0, L>0, M\ge 1$ and $\alpha\in S$ with $|\alpha|>2M+1$ such that for each x, $y\in D$*

$$F(x^*)=0,\ F'(x^*)\ne 0, \tag{10}$$

$$|F'(x^*)^{-1}(F'(x)-F'(x^*))|\le L_0|x-x^*|, \tag{11}$$

$$|F'(x^*)^{-1}(F'(x)-F'(y))|\le L|x-y|, \tag{12}$$

$$|F'(x^*)^{-1}F'(x)|\le M, \tag{13}$$

and

$$\bar{U}(x^*, r)\subseteq D,$$

where the radius r is given by (4). Then, sequence $\{x_n\}$ generated for $x_0 \in U(x^*, r) - \{x^*\}$ by method (2) is well defined, remains in $U(x^*, r)$ for each $n = 0, 1, 2, \cdots$ and converges to x^*. Moreover, the following estimates hold

$$|y_n - x^*| \leq g_1(|x_n - x^*|)|x_n - x^*| < |x_n - x^*| < r, \tag{14}$$

and

$$|x_{n+1} - x^*| \leq g_2(|x_n - x^*|)|x_n - x^*| < |x_n - x^*|, \tag{15}$$

where the "g" functions are defined above Theorem (2.1). Furthermore, for $T \in [r, \frac{2}{L_0})$ the limit point x^* is the only solution of equation $F(x) = 0$ in $\bar{U}(x^*, T) \cap D$.

Proof. We shall show estimates (14) and (15) using mathematical induction. By hypothesis $x_0 \in U(x^*, r) - \{x^*\}$, (5) and (11) we get that

$$|F'(x^*)^{-1}(F'(x_0) - F'(x^*))| \leq L_0|x_0 - x^*| < L_0 r < 1. \tag{16}$$

It follows from (16) and the Banach Lemma on invertible functions [2, 5, 15, 20] that $F'(x_0) \neq 0$ and

$$|F'(x_0)^{-1}F'(x^*)| \leq \frac{1}{1 - L_0|x_0 - x^*|}. \tag{17}$$

Hence, y_0 is well defined by the first sub-step of method (2) for $n = 0$. Using (4), (6), (10), (12) and the first sub-step of method (2) for $n = 0$ we get in turn that

$$\begin{aligned} |y_0 - x^*| &\leq |x_0 - x^* - F'(x_0)^{-1}F'(x^*)| \\ &\leq |F'(x_0)^{-1}F'(x^*)|\left|\int_0^1 F'(x^*)^{-1}[F'(x^* + \theta(x_0 - x^*) - F'(x_0)](x_0 - x^*)d\theta\right| \\ &\leq \frac{L|x_0 - x^*|^2}{2(1 - L_0|x_0 - x^*|)} = g_1(|x_0 - x^*|)|x_0 - x^*| < |x_0 - x^*| < r, \end{aligned} \tag{18}$$

which shows (15) for $n = 0$ and $y_0 \in U(x^*, r)$. We can write by (10) that

$$F(x_0) = F(x_0) - F(x^*) = \int_0^1 F(x^* + \theta(x_0 - x^*))(x_0 - x^*)d\theta. \tag{19}$$

Notice that $|x^* + \theta(x_0 - x^*) - x^*| \leq \theta|x_0 - x^*| < r$. Hence, $x^* + \theta(x_0 - x^*) \in U(x^*, r)$. In view of (13) and (19) we get that

$$|F'(x^*)^{-1})F'(x_0)| \leq \left|\int_0^1 F(x^* + \theta(x_0 - x^*))(x_0 - x^*)d\theta\right| \leq M|x_0 - x^*|. \tag{20}$$

Similarly by replacing x_0 by y_0 in (20), we have that

$$|F'(x^*)^{-1})F'(y_0)| \leq M|y_0 - x^*|.$$

Next, we show that $1 + (\alpha - 1)\frac{F(y_0)}{F(x_0)} \neq 0$ for $x_0 \neq x^*$. We get by (11) that

$$\begin{aligned} &|(F'(x^*)(x_0 - x^*))^{-1}[F(x_0) - F(x^*) - F'(x^*)(x_0 - x^*)]| \\ &\leq |x_0 - x^*|^{-1}\frac{L_0}{2}|x_0 - x^*|^2 = \frac{L_0}{2}|x_0 - x^*| < \frac{L_0}{2}r < 1. \end{aligned}$$

Hence, we have that $F(x_0) \neq 0$ and

$$|F'(x_0)^{-1}F'(x^*)| \leq \frac{1}{|x_0 - x^*|(1 - \frac{L_0}{2}|x_0 - x^*|)}. \tag{21}$$

Then, we have by (4), (7), (18) and (21) that

$$|\alpha - 1||\frac{F(y_0)}{F(x_0)}| \leq \frac{|\alpha - 1|M|y_0 - x^*|}{|x_0 - x^*|(1 - \frac{L_0}{2}|x_0 - x^*|)}$$

$$\leq \frac{|\alpha - 1|Mg_1(|x_0 - x^*|)}{1 - \frac{L_0}{2}|x_0 - x^*|} = p_0(|x_0 - x^*|) < p_0(r) < 1. \tag{22}$$

Hence, we get by (22) that

$$|(1 + (\alpha - 1)\frac{F(y_0)}{F(x_0)})^{-1}| \leq \frac{1}{1 - p_0(|x_0 - x^*|)}. \tag{23}$$

We have by (22) the estimate

$$|1 - (\alpha + 3)(\frac{F(y_0)}{F(x_0)}) - (\alpha^2 - 1)(\frac{F(y_0)}{F(x_0)})^2|$$

$$\leq 1 + \frac{|\alpha + 3|Mg_1(|x_0 - x^*|)}{1 - \frac{L_0}{2}|x_0 - x^*|} + \frac{|\alpha^2 - 1|M^2 g_1^2(|x_0 - x^*|)}{(1 - \frac{L_0}{2}|x_0 - x^*|)^2}. \tag{24}$$

It then follows from (4), (8), (23) and (24) that

$$\frac{1}{|\alpha|}\sqrt{A_0} \leq p(|x_0 - x^*|) < p(r) < 1.$$

Hence, we get that

$$(1 \pm \frac{\sqrt{A_0}}{\alpha})^{-1} \leq \frac{1}{1 - p(|x_0 - x^*|)}. \tag{25}$$

We also have that

$$|I - B_0| \leq \frac{1 + \sqrt{A_0}}{1 - p(|x_0 - x^*|)} \leq \frac{1 + |\alpha|p(|x_0 - x^*|)}{1 - p(|x_0 - x^*|)}. \tag{26}$$

Hence, x_1 is well defined by the second sub-step of method (2) for $n = 0$. Then, we have by (4), (9), (17), (18), (20), (22) and (23) that

$$\begin{aligned} |x_1 - x^*| &\leq |x_0 - x^* - F'(x_0))^{-1}F'(x_0)| + |I - B_0||F'(x_0)^{-1}F(x^*)| \\ &\quad \times |F'(x^*))^{-1}F(x_0)| \\ &\leq \frac{L|x_0 - x^*|^2}{2(1 - L_0|x_0 - x^*|} + \frac{(1 + |\alpha|p(|x_0 - x^*|))M|x_0 - x^*|}{|\alpha|(1 - p(|x_0 - x^*|))(1 - L_0|x_0 - x^*|)} \end{aligned}$$

$$= g_2(|x_0 - x^*|)|x_0 - x^*| < |x_0 - x^*| < r, \tag{27}$$

which shows (15) for $n = 0$ and $x_1 \in U(x^*, r)$. By simply replacing x_0, y_0, x_1 by x_k, y_k, x_{k+1} in the preceding estimates we arrive at estimate (14) and (15). Using the estimate $|x_{k+1} - x^*| < |x_k - x^*| < r$, we deduce that $x_{k+1} \in U(x^*, r)$ and $\lim_{k\to\infty} x_k = x^*$. To show the uniqueness part, let $Q = \int_0^1 F'(y^* + \theta(x^* - y^*)d\theta$ for some $y^* \in \bar{U}(x^*, T)$ with $F(y^*) = 0$. Using (11) we get that

$$\begin{aligned} |F'(x^*)^{-1}(Q - F'(x^*))| &\leq \int_0^1 L_0 |y^* + \theta(x^* - y^*) - x^*| d\theta \\ &\leq \int_0^1 (1-\theta)|x^* - y^*| d\theta \leq \frac{L_0}{2} R < 1. \end{aligned} \tag{28}$$

It follows from (28) and the Banach Lemma on invertible functions that Q is invertible. Finally, from the identity $0 = F(x^*) - F(y^*) = Q(x^* - y^*)$, we deduce that $x^* = y^*$. □

Remark 2.1. 1. In view of (11) and the estimate

$$\begin{aligned} \|F'(x^*)^{-1}F'(x)\| &= \|F'(x^*)^{-1}(F'(x) - F'(x^*)) + I\| \\ &\leq 1 + \|F'(x^*)^{-1}(F'(x) - F'(x^*))\| \leq 1 + L_0\|x - x^*\| \end{aligned}$$

condition (13) can be dropped and M can be replaced by

$$M(t) = 1 + L_0 t$$

or by $M(t) = M = 2$ since $t \in [0, \frac{1}{L_0})$.

2.The results obtained here can be used for operators F satisfying autonomous differential equations [2] of the form

$$F'(x) = P(F(x)),$$

where P is a continuous operator. Then, since $F'(x^*) = P(F(x^*)) = P(0)$, we can apply the results without actually knowing x^*. For example, let $F(x) = e^x - 1$. Then, we can choose: $P(x) = x + 1$.

3.The radius r_1 given by (4) was shown by us to be the convergence radius of Newton's method [2, 5]

$$x_{n+1} = x_n - F'(x_n)^{-1}F(x_n) \text{ for each } n = 0, 1, 2, \cdots \tag{29}$$

under the conditions (9) and (10). It follows from (6) and $r < r_1$ that the convergence radius r of the method (2) cannot be larger than the convergence radius r_1 of the second order Newton's method (22). As already noted in [2, 5] r_1 is at least as large as the convergence ball given by Rheinboldt [19]

$$r_R = \frac{2}{3L}.$$

In particular, for $L_0 < L$ we have that

$$r_R < r$$

and

$$\frac{r_R}{r_1} \to \frac{1}{3} \quad as \quad \frac{L_0}{L} \to 0.$$

That is our convergence ball r is at most three times larger than Rheinboldt's. The same value for r_R was given by Traub [20].

4.It is worth noticing that method (2) is not changing when we use the conditions of Theorem 2.1 instead of the stronger conditions used in [14, 16]. Moreover, we can compute the computational order of convergence (COC) defined by

$$\xi = \ln\left(\frac{\|x_{n+1} - x^*\|}{\|x_n - x^*\|}\right) / \ln\left(\frac{\|x_n - x^*\|}{\|x_{n-1} - x^*\|}\right)$$

or the approximate computational order of convergence

$$\xi_1 = \ln\left(\frac{\|x_{n+1} - x_n\|}{\|x_n - x_{n-1}\|}\right) / \ln\left(\frac{\|x_n - x_{n-1}\|}{\|x_{n-1} - x_{n-2}\|}\right).$$

This way we obtain in practice the order of convergence in a way that avoids the bounds involving estimates using estimates higher than the first Fréchet derivative of operator F.

3. Numerical examples

We present two numerical examples in this section.

Example 3.1. Returning back to the motivational example at the introduction of this study, we have $L_0 = L = 146.6629073$, $M = 2$. Then for $\alpha = 6$ the parameters are

$$r_1 = 0.0045,\ r_2 = 0.0009 = r.$$

Example 3.2. Let $D = [-1, 1]$. Define function f of D by

$$f(x) = e^x - 1. \tag{30}$$

Using (30) and $x^* = 0$, we get that $L_0 = e - 1 < L = M = e$. Then for $\alpha = 6$ the parameters are

$$r_1 = 0.3249,\ r_2 = 0.0540 = r.$$

Example 3.3. Let $D = (-\infty, +\infty)$. Define function f of D by

$$f(x) = \sin(x).$$

Then we have for $x^* = 0$ that $L_0 = L = M = 1$. Then for $\alpha = 4$ the parameters are

$$r_1 = 0.6667,\ r_2 = 0.2656 = r.$$

4. Conclusion

We present a local convergence analysis of a fourth order method in order to approximate a solution of an equation in a Banach space setting. Earlier convergence analysis (on the real line) is based on hypotheses up to the third Fréchet-derivative [1]–[20]. In this paper the local convergence analysis is based only on Lipschitz hypotheses of the first Fréchet-derivative. Hence, the applicability of these methods is expanded.

References

[1] Amat, S., Hernández, M. A., Romero,N., (2012), Semilocal convergence of a sixth order iterative method for quadratic equations, Applied Numerical Mathematics, 62, pp.833-841.

[2] Argyros, I. K., (2007), Computational Theory of Iterative Methods. Series: Studies in Computational Mathematics, 15, Editors: C.K.Chui and L. Wuytack, Elsevier Publ. Co. New York, U.S.A.

[3] Argyros, I. K., (2011), A semilocal convergence analysis for directional Newton methods. Math.Comput., 80, pp.327–343.

[4] Argyros, I. K., Hilout. S., (2012), Weaker conditions for the convergence of Newton's method. J. Complexity, 28, pp.364–387.

[5] Argyros, I. K., Hilout, S., (2013), Computational Methods in Nonlinear Analysis. Efficient Algorithms, Fixed Point Theory and Applications, World Scientific.

[6] Argyros, I.K., Ren, H., (2012), Improved local analysis for certain class of iterative methods with cubic convergence, Numerical Algorithms, 59, pp.505-521.

[7] Chun, C., Lee, M. Y., Neta, B., Dzunic, J., (2012), On optimal fourth order iterative methods free from second derivative and their dynamics, Appl. Math. Comput., 218, pp.6427–6438.

[8] Cordero, A., Torregrosa, J. R., Vasileva, M. P., (2013), Increasing the order of convergence of iterative schemes for solving nonlinear systems, J. Comput. Appl. MAth., 252, pp.86–94.

[9] Ezquerro, J. A., Gonzalez,D., Hernández, M. A., (2013), A modification of the classic conditions of Newton-Kantorovich for Newtons method, Math. Comput. Model., 57, pp.584–594.

[10] Ezquerro, J. A., Gonzalez, D., Hernández, M. A., (2012), A variant of the Newton-Kantorovich theorem for nonlinear integral equations of mixed Hammerstein type, Appl. Math. Comput., 218, pp.9536–9546.

[11] Gutiérrez, J. M., Magreñán, A. A., Romero, N., (2013), On the semi-local convergence of Newton-Kantorovich method under center-Lipschitz conditions, Applied Mathematics and Computation, 221, pp.79–88.

[12] Hansen, E., Patrick, M., (1977), A family of root finding methods, Numer. Math. 27, pp.257–269.

[13] Hernández, M. A., Salanova, M. A., (2000), Modification of the Kantorovich assumptions for semi-local convergence of the Chebyshev method, Journal of Computational and Applied Mathematics, 126, pp.131-143.

[14] Kansal, M., Kanwar. V., Bhatia, S., (2015), New modifications of Hansen-Patrick's family with optimal fourth and eighth orders of convergence, Appl. Math. Comput., 269(15), pp.507–519.

[15] Kantorovich, L. V., Akilov, G. P., (1982), Functional Analysis, Pergamon Press, Oxford.

[16] Kou, J. S., Li. Y. T., Wang, X. H., (2006), A modification of Newton method with third-order convergence, Appl. Math. Comput., 181, pp.1106–1111.

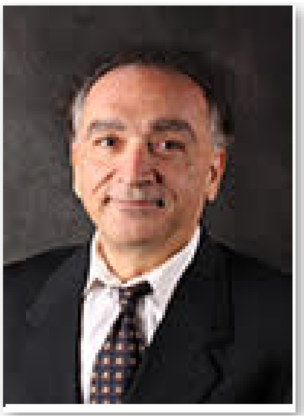
[17] Magrenan, A. A., (2014), Different anomalies in a Jarratt family of iterative root finding methods, Appl. Math. Comput., 233, pp.29-38.

[18] Petkovic, M. S., Neta, B., Petkovic, L., Džunič, J., (2013), Multipoint Methods for Solving Nonlinear Equations, Elsevier.

[19] Rheinboldt, W. C., (1977), An adaptive continuation process for solving systems of nonlinear equations, In: Mathematical models and numerical methods (A.N.Tikhonov et al. eds.) pub.3, 129-142 Banach Center, Warsaw Poland.

[20] Traub, J. F., (1982), Iterative Methods for the Solution of Equations, AMS Chelsea Publishing.

[21] Tuğ, O., Başar, F., (2016), On the spaces of Nörlund null and Nörlund convergent sequences, TWMS J. Pure Appl. Math., 7(1), pp.76–87.



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